**Max-Min Model**

**Introduction**

Controlling risk and quantifying losses are, in managing a portfolio, two fundamental problems that an investor would easily want to solve. This process is very difficult to quantify accurately, and even investment management companies have focused on the importance of efficient risk management within the portfolio.

The goal of improving the performance of the Markowitz model has led many researchers to formulate different portfolio optimization models with the introduction of new alternative measures of risk.

In fact, we present the **Max Min model (MM) – Young, 1988.**

**Max-Min Model**

To find the optimal solution, the minimum portfolio return rather than the variance (as per Markowitz) is defined as the measure of risk. This model attempts to show that this asymmetric measure of risk is more appropriate for asymmetric distributions of returns.

In particular, the portfolio is selected that minimizes the maximum possible loss with respect to all past periods, and with the restriction of a certain minimum average return level defined as acceptable throughout the observation period.

This model is also known as the Minimax model since maximizing the minimum portfolio return corresponds to minimizing the maximum loss.

First, suppose we have the observed historical data for 𝑁 securities, in the time period 𝑡 = 1,…, 𝑇, then we can define the following variables:

***Xj*** portfolio fraction invested in j, j=1,2,...,n

***Rjt*** asset j return at time t, j=1,2,...,n; t=1,2,...,T

***E(Rj)*** asset j expected return, j=1,2,...,n

***R\**** fixed value for the portfolio expected return

***Z*** minimum portfolio return

The risk of a portfolio (𝑋1, 𝑋2,..., 𝑋𝑛) is evaluated by measuring the minimum return of the portfolio observed in the past time periods 𝑡 = 1,...,𝑇.

The MaxMin portfolio maximizes the amount *Z*, subject to the restriction that the average return *𝐸(Rj)* exceeds a certain threshold, desirable by the investor and that the sum of the capital invested in each individual security does not exceed the budget, in this case equal to 1.

**Formulation:**

1. Let z be the (unknown) minimum
2. Replace z in the model o.f.
3. Insert 𝑻 **constraints** guaranteeing that 𝑧 is the minimum

**Manca la formula che non so da dove l’avete presa**

**R min**

represents the minimum return maximized that we can take (it can also be seen as a maximum loss minimized by changing the sign)

We firstly calculated the Expected Return with respect to each asset by doing the average of each one for the whole time period.

The expected return is the profit or loss that an [investor](https://www.investopedia.com/terms/i/investor.asp) anticipates on an investment that has known historical [rates of return](https://www.investopedia.com/terms/r/rateofreturn.asp) (RoR).

We can calculate the Expected Return as:

Then, we proceed to set the decision variables as fraction of the capital Xi invested in the 30 assets.

At this point we started to impose/implement/add the constraints to our model, starting from the portfolio constraint in which we set that the sum of the weight of the assets must be equal to 1.

Finally, to **maximize the minimum loss,** we need to define Z as the minimum value of the weighted return:

1. The next constraint to be imposed is the Z Value Constraint, the Z Value is calculated by taking the minimum value of the sum of each return multiplied by their weight for each time.
2. Once we found the Z Value, we imposed that the Z Value (which is in the RHS) must be lower or equal (<=) than the LHS.

We use the Excel Solver to maximize the objective function (z), then we set the assets weight as the decision variables and by adding all the previous mentioned constraints, we maximize the objective function (as a linear model).

Once we use the Excel Solver we can see a variation in the values of the weights of our assets.

**R max:**

Now our purpose is to find out the maximum expected return (Rmax) of the portfolio,

1. by multiplying the weight of each asset per the expected return of each asset
2. In this case the only constraint this the one related to the portfolio, which tell us that LHS = RHS. (quello che avevi commentato te Rami)

We used the Excel solver to maximize the objective function (expected return of the portfolio), then we set the assets weight as the decision variables and by adding the portfolio constraint, we maximize the objective function (as a linear model).

**Efficient frontiers:**

when choosing a portfolio, the investor prefers a high return and a low risk.

Given the above principle, it is possible to reduce the set of possible portfolios to a smaller subset by eliminating those portfolios which are surely worse than another one available in the same market.

According to the principle we know that the investor would like to pick a portfolio which is located in the region in which return increases and risk decreases

The expected return constraint that we use, is given by imposing the sum of the weight per their expected return equal to the target value

The purpose of this step is to fix 10 equally spaced values between Rmin and Rmax and compute the MM Model, by doing so, every time we want to find the MinRisk value (associated to each value), we change the expected return constraint and maximize z

In this way we can plot the EF, related to the minimum return maximized against the maximized expected return, by plottng the graph we should expect the results in the Northwest box.

Instead, if we want to analyze it from a loss point of view, we can change the sign of our minimum return maximized so that we have a loss but the plot as we are used to.

As we can see in the plot we have a increase of 10% between each point starting from Rmin to P10 and then a bigger gap between our P10 and Rmax, obviously because the difference between this last two points, is higher than 10%

**Cardinality and buy-in threshold:**

We are going to introduce the Cardinality and Buy-in threshold constraints into our model, considering just the period up to end of 2005.

By Mathematical Programming we are able to model conditions on how many and which assets are included in the selected portfolio. We can fix a precise or a maximum portfolio cardinality.

This model is more sophisticated than the revious ones, siince we need additional binaryvariables 𝑌 , 𝑖 = 1,2,...,𝑛.

We define:

1 if asset is included in the portfolio

0 otherwise

Yi

Logical constraint modelling the dependance between variables X and Y

Shape

Description automatically generated

**Buy-in threshold constraints**

By exploiting binary variables, buy-in threshold constraints can be formulated to model a double decision:

1) selecting or not an asset;

2) bounding the fraction invested in a selected asset.

We still need the additional binary variables 𝑌 , 𝑖 = 1,2, . . . , 𝑛: 𝑖

Diagram

Description automatically generated

**Non so cosa mettere di questo pezzo**

The program becomes a Quadratic Mixed Integer Program, larger in size and computationally more complex.

Due to the precence of the new binary variables 𝑌, 𝑖 = 1,2,...,𝑛, these

constraints are frequently included together with cardinality constraints.

The Cardinality constraint set the limit of assets that we should include in our portfolio to 5, while the Buy-in threshold determines a range (between 0.1 < Xi < 0.4) in which our assets must be included

With regard to the buy-in threshold we faced the problem concerned the excel solver, which has a maximum capacity of 200 decision variable cells and 100 constratints cells. For that reason we could not include the upper bound constraint (0.4) into the solver. In any case, in our portfolio we did not have any assets that exceeded the upper bound, so we could say that the constraint was respected.

By doing the cardinalityconstraint, we set 30 new binary variables which can only take either 0 (is the asset is not included) or 1 (if the asset i included) as value. In order to activate them, we also imposed another constraint which is given by (the formula). Finally we imposed the sum of the Yi variables less or equal to 5.

For the Buy-in threshold we imposed that LB < Xi < UB. In order to calculate the bounds, we have to multiply the values of the Yi per the thresholds that were given.

The last constraint to be considered, concerned the target value, which should be imposed as the expected return. The target was defined as 1/2(Rmin+Rmax)

We use the Excel Solver in order to maximize the Objective Function, and thus we found the optimal portfolio up to the end of 2005 and will be later used to compute the returns up to 11/2006.

As expected our optimal portfolio includes 5 assets that respect all of our constraints.